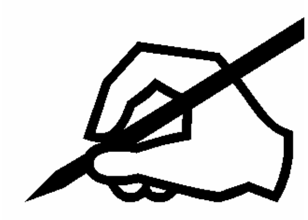

LESSON SUMMARY

CXC CSEC MATHEMATICS

UNIT TWO:
COMPUTATION

Lesson

3

Approximating and Representing Real Numbers

Textbook: Mathematics, A Complete Course by Raymond Toolsie, Volume 1

(Some helpful exercises and page numbers are given throughout the lesson e.g. Ex 3j page 66)

INTRODUCTION

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In this lesson we will look at approximating real numbers. We will also look at writing numbers in standard form and the average of a set of real numbers. These techniques are important because it allows for the representation of numbers when the exact values may be impractical. Ratio and proportions which are two important computational skills that are used to compare real values are also dealt with here.

OBJECTIVES

At the end of this lesson you will be able to:

a) approximate a value to a given number of significant figures or decimal places;
b) write rational numbers in standard form;
c) compare two quantities in a given ratio;
d) divide a given quantity in a ratio;
e) solve problems involving ratio and proportions.

2.5 Approximation

When you approximate a value you get something close to the value but not equal to it.

Approximation to the nearest whole number

To approximate to the nearest whole number, put a cut off line on the decimal point. If the digit just after the cut off line is 5 or more then add 1 to the digit that is just before the cut off line. If it is less than 5 do not add 1. Remove any digits after the cut off line.

Example: Write the following decimal number correct to the nearest whole number: 74. 53.

This means 'is approximately equal to'.

Solution: 74.|53 ≈ 75.

5 therefore add 1 to the number just before the cut off line.

Approximation to the nearest power of ten

To approximate a number to the nearest power of ten put the cut off line just after the place value you are approximating to. Look at the digit to the right of the cut off line, if it is 5 or more add 1 to the digit just before the cut of line. If it is less than 5 do not add 1. Replace any digit after the cut off line with zeros.

Example: Write the following number as an approximate number of hundreds: 8427.

Zeros are put to replace the 2 and 7.

Solution: 84|27 ≈ 8400.

This is less than 5 therefore do not add 1to the digit before the cut off line.

ACTIVITY 1

State each of the following numbers correct to the nearest number of tens and hence determine an approximate answer for each problem. (Ex 3j page 66).

(i) $314 - 82$
(ii) $751 + 349 - 463.$

Approximation to a given number of decimal places

To express a number to a given number of decimal places count the required number of places after the decimal point and place a cut off line just after it. Look at the digit to the right of the cut off line, if it is 5 or more add 1 to the digit just before the cut of line. If it is less than 5 do not add 1. Remove any digits after the cut off line.

Example: State the following number correct to the number of decimal places given in brackets: 286.598 (2 d.p.).

This is more than 5 therefore add 1 to the digit to the left of the cut off line. This digit is however a nine so it becomes 0 and the digit before it is raised by 1.

Solution: 286.59|8 $\approx 286.60.$

Approximation to a given number of significant figures

The important thing to remember is that 0 can be a significant figure however it cannot be the first significant figure. Do not confuse significant figures with decimal places.

To express a number to a given number of significant figures, count the required number of significant figures (do not start with a 0) and place a cut off line just after it. Look at the digit to the right of the cut off line, if it is 5 or more add 1 to the digit just before the cut of line. If it is less than 5 do not add 1. Replace any digit after the cut off line with zeros. You need not put zeros to replace digits after the decimal point.

Example: Express the number 0.00050849 correct to 3 significant figures.

Solution: 0.000508|49 = 0.000508

First significant figure.

ACTIVITY 2

Write 506.00679: (Ex 3k page 68)

(i) Correct to 4 decimal places.

(ii) Correct to 4 significant figures.

2.6 Standard Form or Scientific Notation

Writing a rational number in standard form means putting it in the form, $a \times 10^n$.
NB. 'a' must be greater than or equal to 1 but less than 10, i.e. $1 \leq a < 10$. Also 'n' must be an integer.

Obtaining 'a' involves moving the decimal point to the left or the right as required. If the decimal point is moved to the left 'n' is positive, if it is moved to the right 'n' is negative.

Example: Write 0.0007308 in standard form.

Solution: 0.0007.308

a = 7.308

$n = -4$ since the decimal point was moved 4 places to the right.

Therefore the standard form for 0.0007308 is 7.308×10^{-4}. If the standard form is expanded we get back the original number.

ACTIVITY 3

Express each of the following numbers in standard form: (Ex 3m page71)

(i) 3 850 000 000.

(ii) 0. 000 0437

2.7 Ratio and Proportions

A ratio compares the size of two quantities.

Example:

The ratio of ☆ to ♡ is 4:6 = 2:3. The ratio of ☆ to the total number of shapes is 4:10 = 2:5. The ':' means to.

A ratio can be written as a fraction. Therefore 2:5 can be written as $\frac{2}{5}$.

Example: Express $6:30¢ as a ratio in its lowest terms. (Ex 3p page 78).

Solution: $6 is 600 cents therefore 600:30 = $\frac{\cancel{600}^{20}}{\cancel{30}_{1}}$ = 20:1.

Proportions are equal ratios.

Example: ○ □ ○ □ ○

The ratio of □ to ○ is 2:3. This is the same ratio for ☆ to ♡ . Therefore the two sets of shape are in proportion. The following examples show how the concept of proportions may be applied.

Example: $15 000 is to be shared between Mary and Jane in the ratio 2:3 respectively. How much money will Jane receive?

Solution: There are 2 + 3 = 5 parts. Jane will get 3 parts.

The amount of money Jane will receive is $\frac{3}{5} of\ \$15000 =$

$\frac{3}{\cancel{5}_{1}} \times \$1\cancel{5000}^{\$3000} =$ $9000.

Example: two lengths are in the ratio 7:8. If the first length is 273m, what amount is the second length?

Solution: set up a proportion, $\frac{7}{8} = \frac{273m}{second\ length}$

$7 \times second\ length = 273m \times 8$

$$second\ length = \frac{273m \times 8}{7} = 312\text{m}$$

ACTIVITY 4

(Ex 3q page 80 and Ex 3r 81)

1. An estate valued at $75,000 is divided among three daughters, Natasha, Natalie and Nadia in the ratio 5:8:2 respectively. Calculate the amount each receives.

2. Two amounts of money are in the ratio 8:3. If the second amount is $4.05, what is the value of the first amount?

2.8 Averages

This is also called the Arithmetic Mean. You can find the average of a set of quantities by adding up all the quantities and dividing by the number of quantities.

Example: Erica's marks in eight consecutive Mathematics Examinations were: 94, 83, 75, 52, 71, 68, 75, 49. What was her average mark?

Solution: $\frac{94+83+75+52+71+68+75+49}{8} = \frac{567}{8}$

Average = 70.875

You can use the average to find the sum of a set of quantities by simply multiplying it by the number of quantities.

Example: Beverly's average mark for eight examination papers was 74.5. How many marks did she score altogether. (Ex 3v page91)

Solution: altogether she scored $74.5 \times 8 = 596$

ASSESSMENT

Answer the following questions. The first one is a multiple choice item.

1. 0.045×10^{-3} in scientific notation is

   (A) 4.5×10^{-6}

   (B) 4.5×10^{-5}

   (C) 4.5×10^{-4}

   (D) 4.5×10^{-1}

2. Write the value of 0.428×2.75 :

   (i) exactly in decimal form

   (ii) to two decimal places

   (iii) to two significant figures

CONCLUSION

As mentioned at the start of the lesson, being able to represent numbers is an extremely important skill in Mathematics. It is important because Mathematics is about communicating results of computation in an efficient manner. The next unit deals with Set Theory. Operations that involve the manipulation of sets of numbers will be further developed.